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Periodic, fully developed, natural convection in a channel with corrugated confining walls

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Abstract—Numerical solutions are obtained for natural convection heat transfer in an open channel with corrugated, isothermal confining walls. The channel is very long so that the fluid temperature approaches the wall temperature and the flow can be assumed to be periodically fully developed. The solutions are obtained by solving the full elliptic governing equations in a transformed coordinate system which maps the channel with corrugated walls onto a channel with flat walls. The periodic, fully developed Nusselt number for the corrugated channel is expressed by the relation $Nu = C\,Gr\,Pr/(L/W)$ where Gr, Pr and L/W are the Grashof number, the Prandtl number and the aspect ratio, respectively, and C is a parameter which is a function of Gr, L/W and the corrugation angle θ. In the limiting case of $\theta = 0°$ (two flat walls), the parameter C approaches a constant value. This value is within 1.6% of the exact analytical result.

INTRODUCTION

NATURAL convection heat transfer between parallel plates has been studied extensively since the initial analysis of Bodoia and Osterle [1] (see [2] for a representative bibliography). However, with the exception of a recent paper by Agonafar and Watkins [3], who treated natural convection between diverging plates, there is no single treatment of natural convection in open channels with irregular side walls. This has motivated the present authors to investigate, seemingly for the first time, periodic, fully developed, natural convection heat transfer between two corrugated walls.

The solution methodology that was developed in a previous paper [4] to solve the forced convection problem in a corrugated channel was adopted in the present investigation. The basis of the method is an algebraic coordinate transformation which maps the complex domain onto a rectangle. The numerical solutions were performed for laminar flow and for thermal boundary conditions of uniform wall temperature. The calculations were carried out for a number of aspect ratios L/W, for three values of the corrugation angle θ, and for a range of Grashof numbers Gr.

FORMULATION

Description of the problem

The physical situation to be investigated—through numerical solutions of the conservation equations—is a corrugated, vertical, open channel as depicted schematically in Fig. 1(a). The geometry of the channel is specified by the axial length of one cycle L, the horizontal spacing between corrugated walls W and the corrugation angle θ. The sharp-edged corrugation corners are approximated by smooth curves to prevent unrealistic solutions. The half length of the chord of this curve is denoted by l and is assumed to be a small number in the present problem (i.e. $l = L/256$). If the left and the right walls are positioned such that the peaks of both lie in the same plane, then the width W is a function of θ and L and can be expressed as $W = (L/2)\tan\theta$. Then the selected values of L/W range from $1/\tan\theta$ to $4/\tan\theta$.

The solution domain, with the assumption of a periodic, fully developed flow, is confined to a typical module shown in Fig. 1(b). As is described by Patankar [5], the periodic, fully developed flow is characterized by a velocity field that repeats itself at corresponding axial stations in successive cycles. Furthermore, in such a regime, the pressure decreases linearly in the downstream direction at corresponding periodic locations. Similarly, a periodic, thermally

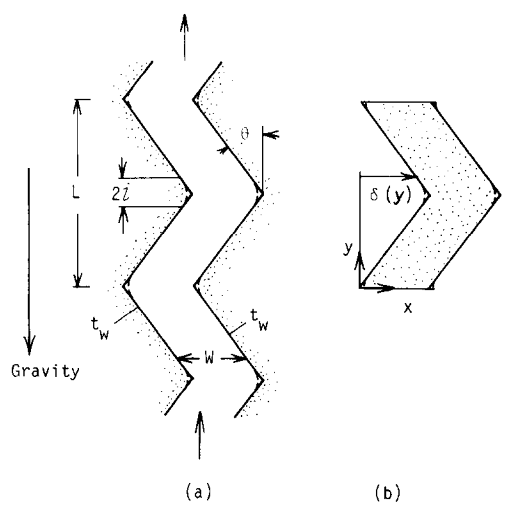

FIG. 1. (a) Schematic diagram of the corrugated wall channel. (b) A periodic module.

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NOMENCLATURE

A_w	per cycle area
C	defined by equation (15)
c	specific heat
g	acceleration due to gravity
Gr	Grashof number as defined in equation (1)
h	periodic, fully developed heat transfer coefficient
K	thermal conductivity of the fluid
L	axial length of a cycle
P	dimensionless periodic pressure
p'	periodic pressure
Pr	Prandtl number
Q	heat transfer rate
t	temperature
T	dimensionless temperature
U, V	dimensionless velocity components
u, v	velocity components
W	horizontal spacing between corrugated walls
X	dimensionless transverse coordinate, x/W
x	transverse coordinate
y	coordinate along the streamwise direction
Y	dimensionless streamwise direction, y/W.

Greek symbols

$\delta(y)$	width of the solution domain
β	thermal expansion coefficient
η	transformed coordinate, equation (7)
θ	corrugation angle
μ	viscosity
ν	kinematic viscosity
ξ	transformed coordinate, equation (7)
ρ	density
ψ	streamfunction
Ψ	dimensionless streamfunction.

Subscripts

w	refers to the wall
∞	refers to the free-stream values
ent	refers to entrance
exit	refers to exit.

developed regime exists for a boundary condition of uniform wall temperature. In this case, the cycle-averaged heat transfer coefficient does not change in successive cycles.

Attention will now be turned to the conservation equations which describe the flow and heat transfer characteristics for natural convection heat transfer in a corrugated, vertical channel, with the assumption of a periodic, fully developed regime.

The conservation equations

The governing equations employed here are the same as those used in the analysis of natural convection channel flows. In these equations, the streamwise second derivatives and the pressure variations transverse to the streamwise direction are also present. Laminar flow is assumed to prevail, and the only fluid property variation considered is the density difference needed to establish the buoyancy term, for which the Boussinesq density–temperature approximation is employed. The following dimensionless variables are used:

$$X = x/W, \qquad Y = y/W,$$
$$U = u/(\nu/W), \qquad V = v/(\nu/W)$$
$$P = p'/\rho(\nu/W)^2, \qquad T = (t-t_\infty)/(t_w-t_\infty),$$
$$Gr = g\beta W^3(t_w-t_\infty)/\nu^2 \tag{1}$$

where W is the horizontal spacing between the corrugated walls, p' is the pressure difference between the local values within the duct and the ambient values at the same elevation. Then, upon introduction of the dimensionless variables, the governing equations have the following forms:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X}+\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2} \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y}+\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}+Gr\,T \tag{3}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right). \tag{4}$$

With the assumption of a fully developed velocity and temperature, the velocity behaves in a periodic manner from module to module, and the non-dimensional temperature T becomes 1. Then, equations (2) and (3) become

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X}+\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2} \tag{5}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y}+\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}+Gr. \tag{6}$$

From the examination of the governing equations (5) and (6), it can be seen that there is only one parameter whose value has to be specified prior to the initiation of the numerical solutions. This is the Grashof number, Gr. The selected values for Gr are 100, 1000, 10 000 and 100 000. Aside from Gr, there are two dimensionless geometric parameters which have to be specified. These are the corrugation angle θ and the aspect ratio L/W. The values of 0°, 15°, 30°, and 45°

are assigned for θ and the values of L/W range from $1/\tan\theta$ to $4/\tan\theta$.

Analytical and numerical methods

A simple algebraic coordinate transformation is introduced which maps the physical domain onto a rectangle. Specifically, the x, y coordinates are transformed to η and ξ coordinates by the relation

$$\eta = [x-\delta(y)]/W, \quad \xi = y/W \tag{7}$$

such that $\eta = 0$ and 1 at all points on the left and the right walls, respectively. In terms of the new coordinates, the solution domain is defined by $0 < \eta < 1$, and $0 < \xi < L/W$.

The exact analytical expressions for $\delta(Y)$ and its derivatives, and also the transformed equations and their discretization and solutions are documented in an earlier paper [4]. The discretized procedure of the transformed equations is based on the power-law scheme of Patankar [6], and the discretized equations are computed by using a line-by-line method. The pressure is computed by adopting the SIMPLE algorithm of Patankar [6].

The computations were performed with (18×34) grid points. The 18 grid points utilized in the η direction, were distributed in a nonuniform manner with high concentration of grids close to the walls. Supplementary runs were performed with (12×22) and (26×50) grid points to investigate grid size effects for the case of $\theta = 30^\circ$ and $L/W = 3.464$. The changes in the mass flow rates $[\psi_w = \int_0^W (\rho u)_{y=0}\,dx]$ between the coarse mesh (12×22) and the medium mesh (18×34) were 2.7%, 1.2%, 1.3% and 9.0%, and between the medium mesh (18×34) and the fine mesh (26×50) were 0.95%, 2.3%, 0.6% and 11% at $Gr = 10^2$, 10^3, 10^4 and 10^5, respectively. Thus, the medium mesh (18×34) was chosen to maintain relatively moderate computer cost.

Nusselt number

Attention will be directed to obtain an expression for the Nusselt number for periodic, fully developed flow in a corrugated duct. By definition

$$Nu = hW/K \tag{8}$$

$$h = Q/[A_w(t_w - t_{ent})] \tag{9}$$

and A_w is the per-cycle heat transfer area, approximately equal to $2L/\cos\theta$, Q is the rate of heat transfer from both walls to the fluid per cycle and is given by

$$Q = \int_{exit} \rho uct\,dx - \int_{ent} \rho uct\,dx. \tag{10}$$

This equation can be re-written by defining streamfunction as $\psi_w = \int \rho u\,dx$ and assuming $t_{exit} = t_w$ for a long pipe to give

$$Q = c(t_w - t_{ent})\psi_w \tag{11}$$

substituting equations (11) and (9) into (8), the Nusselt number can be expressed as

$$Nu = \psi_w Pr(W/A_w) \tag{12}$$

where ψ_w is a dimensionless streamfunction defined as

$$\psi_w = \psi_w/\mu. \tag{13}$$

Equation (12) can further be expressed as

$$Nu = C\,Gr\,Pr/(L/W) \tag{14}$$

where

$$C = \psi_w/[Gr(A_w/L)]. \tag{15}$$

In the case of $\theta = 0^\circ$, the terms $\partial/\partial Y$, $\partial^2/\partial Y^2$ in equation (6) become zero and the momentum equation reduces to

$$\partial^2 V/\partial X^2 = -Gr. \tag{16}$$

Integrating this equation twice, one obtains V as

$$V = (Gr/2)(X - X^2). \tag{17}$$

Therefore, the streamfunction ψ_w can be obtained as

$$\psi_w = \int_0^1 V\,dX = (1/12)\,Gr \tag{18}$$

and the value of C from equation (15) turns out to be 1/24.

RESULTS AND DISCUSSION

Representative streamline maps obtained from the solution for $\theta = 30^\circ$ are presented in Figs. 2–4. These figures are for $L/W = 2.389$, 3.464 and 6.928, respectively, and in the Grashof number range from 10^2 to 10^5. As seen from these figures, at low Grashof number, there are no separation bubbles at the corners. As the Grashof number is increased, separation regions are observed. This effect is more pronounced at low aspect ratio L/W.

The nondimensional streamfunction is plotted as a function of the Grashof number in Fig. 5 with θ and L/W as curve parameters. An analytical value of the nondimensional streamfunction for the limiting case of $\theta = 0^\circ$ from equation (18) is also plotted in this figure. The nondimensional streamfunction ψ_w represents the nondimensional mass flow by its definition [equation (13)]. The mass flow increases with the Grashof number as is expected, but it is always less than that for the limiting case of $\theta = 0^\circ$ (two flat walls). This is because the friction loss increases and the mass flow decreases with the corrugation angle θ. When θ is 30° and 45°, the mass flow takes a minimum value somewhere in the middle range of the parameter values of L/W.

The coefficient C is plotted as a function of the Grashof number in Fig. 6 with θ and L/W as curve parameters. Its analytical value for the limiting case of $\theta = 0^\circ$ is 1/24 and is independent of the Grashof number. However, it is a function of the Grashof

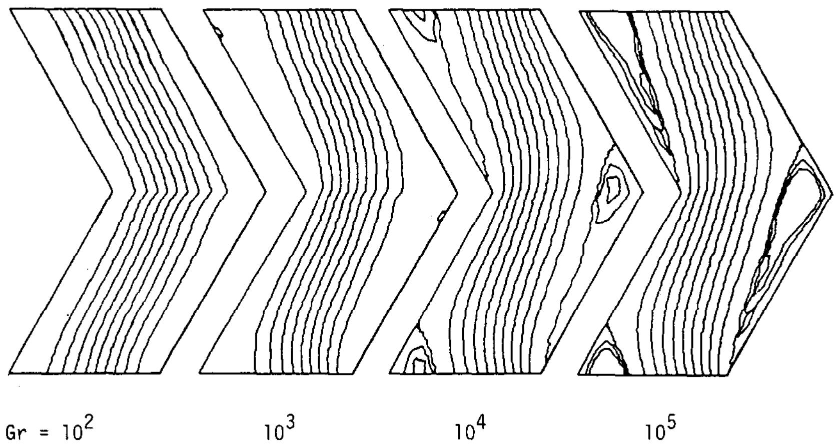

FIG. 2. Streamline plots for $\theta = 30°$ and $L/W = 2.389$.

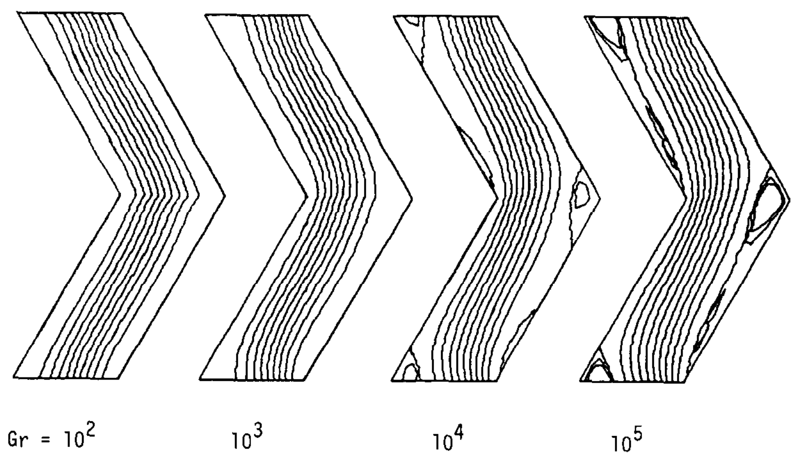

FIG. 3. Streamline plots for $\theta = 30°$ and $L/W = 3.464$.

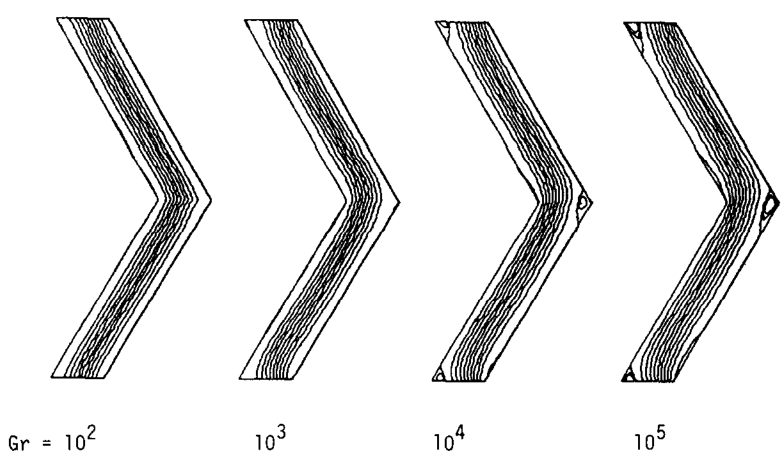

FIG. 4. Streamline plots for $\theta = 30°$ and $L/W = 6.928$.

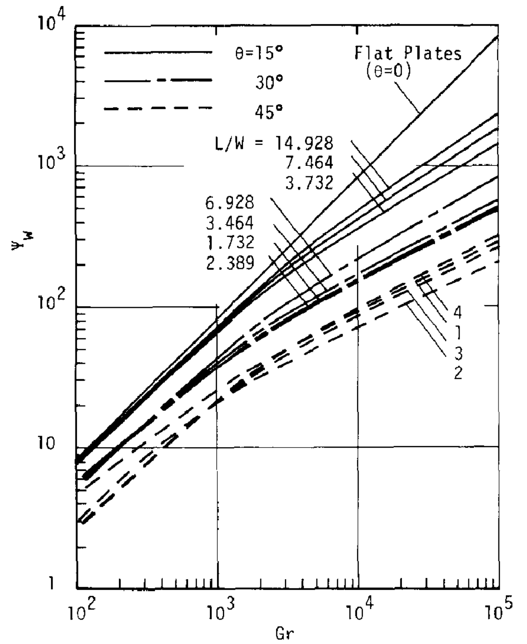

FIG. 5. Nondimensional streamfunction as a function of Grashof number.

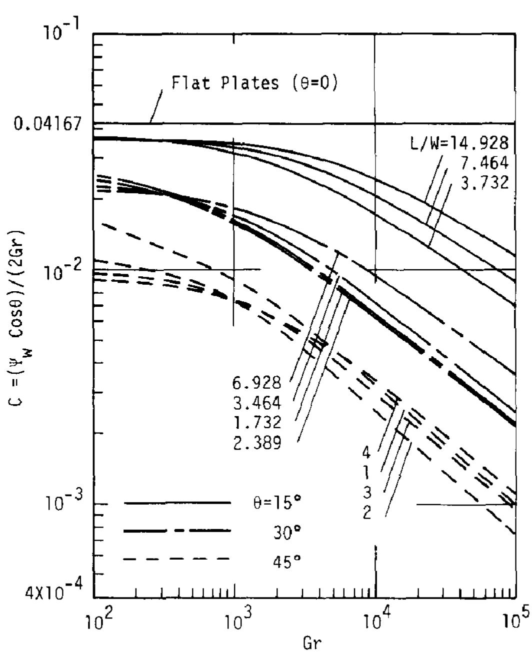

FIG. 6. Coefficient C as a function of Grashof number.

Table 1

Gr	ψ_w	Error (%)	C	Error (%)
100	8.4625	1.6	0.042313	1.6
1000	84.625	1.6	0.042313	1.6
10 000	843.86	1.3	0.042193	1.3
100 000	8457.2	1.5	0.042286	1.5

number and the geometry of the duct for a corrugated channel. This coefficient is always less than 0.04167 and decreases with the Grashof number and increases with the duct angle θ. It takes a minimum value somewhere in the middle range of the parameter value L/W, for the cases of $\theta = 30°$ and $45°$.

Finally supplementary runs are performed with (18×34) grid points to compare ψ_w and C with the analytical values for the case $\theta = 0°$, as given by Table 1. The values agree to within 1.6%.

CONCLUDING REMARKS

The periodic, fully developed results are obtained for natural convection heat transfer in an open channel with corrugated confining walls. The solutions were obtained by finite-difference technique and via utilization of a coordinate transformation methodology. The periodic, fully developed Nusselt number was expressed in terms of Grashof number Gr, Prandtl number Pr, aspect ratio L/W and a parameter which is a function of Gr, L/W and θ. The results agreed satisfactorily with analytical values for the case of $\theta = 0°$.

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CONVECTION NATURELLE PERIODIQUE, ETABLIE DANS UN CANAL AVEC DES PAROIS CORRUGUEES

Résumé—On obtient des solutions numériques pour la convection thermique naturelle dans un canal avec des parois corruguées, isothermes. Le canal est très long de telle façon que la température du fluide approche la température de la paroi et que l'écoulement puisse être supposé périodique et établi. Les solutions sont obtenues en résolvant les équations elliptiques du problème dans un système de coordonnées transformées qui représente le canal avec corrugations comme un canal à parois planes. Le nombre de Nusselt périodique pleinement établi pour le canal corrugué est exprimé par la relation $Nu = C\,Gr\,Pr/(L/W)$ où Gr, Pr et L/W sont les nombres de Grashof et de Prandtl et le rapport de forme et où C est un paramètre fonction de Gr, L/W et de l'angle de corrugation θ. Dans le cas limite $\theta = 0°$ (deux parois planes), le paramètre C approche une valeur constante. Cette valeur est égale à 1,6% près à la solution analytique exacte.

PERIODISCHE, VOLLENTWICKELTE, NATÜRLICHE KONVEKTION IN EINEM KANAL MIT GEWELLTEN WÄNDEN

Zusammenfassung—Es wurden numerische Lösungen für den Wärmeübergang bei natürlicher Konvektion in einem offenen Kanal mit isothermen, gewellten Wänden ermittelt. Da der Kanal sehr lang ist, erreicht die Fluidtemperatur annähernd die Wandtemperatur. Daher kann angenommen werden, daß es sich um eine periodische, voll entwickelte Strömung handelt. Nach einer Koordinatentransformation wurde das vollständige elliptische Gleichungssystem gelöst; dabei wurde der Kanal mit den gewellten Wänden auf einen Kanal mit flachen Wänden abgebildet. Die periodische, voll entwickelte Nusselt-Zahl für den gewellten Kanal wird durch die Beziehung $Nu = C\,Gr\,Pr/(L/W)$ korreliert, wobei L/W das Längenverhältnis und C einen Parameter, der eine Funktion von Gr, L/W und vom Wellungswinkel θ ist, darstellt. Im Grenzfall für $\theta = 0°$ (2 ebene Wände) nähert sich der Parameter C einem konstanten Wert. Dieser Wert hat eine Abweichung von weniger als 1,6% von der exakten analytischen Lösung.

ПЕРИОДИЧЕСКАЯ, ПОЛНОСТЬЮ РАЗВИТАЯ ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В КАНАЛЕ С ГОФРИРОВАННЫМИ ОГРАНИЧЕННЫМИ СТЕНКАМИ

Аннотация—Получены численные решения для теплообмена естественной конвекцией в открытом канале с гофрированными, изотермическими ограниченными стенками. Канал имеет большую длину, таким образом температура жидкости приближается к температуре стенки и течение может считаться периодически полностью развитым. Решения получены с помощью полной системы определяющих уравнений эллиптического типа в трансформированной системе координат, которая отображает канал с гофрированными стенками на область с плоскими стенками. Число Нуссельта для периодического полностью развитого течения в гофрированном канале выражено зависимостью $Nu = CGrPr/(L/W)$, где Gr, Pr и L/W—числа Грасгофа, Прандтля и отношение длины к высоте, соответственно, а C—параметр, являющийся функцией Gr, L/W и угла гофрирования θ. В предельном случае, когда $\theta = 0°$ (две плоские стенки), параметр C приближается к постоянному значению. Это значение отличается на 1,6% от точного аналитического результата.